Exam 1 Solutions & Feedback Key

General Feedback Key

I use this key to concisely and consistently mark common comments on your work. In interpreting a mark, please compare your answer to the solutions below. If you'd like further clarification of what I mean in a particular case, you're welcome to drop by office hours.

- (AT) **Anything implies true.** Starting with a statement that you want to prove, and then showing that you can derive a true statement from it, does not prove anything about the truth of your original statement.
- (C) Please be more **concise**. This mark may also indicate extraneous text.
- (CX) This is needlessly **complicated**. Simpler approaches or arguments exist that are equally good, but easier to convey.

  In particular, note that in CS202, part of the *subject matter of the course* is which proof and problem-solving techniques are appropriate or preferable in which situations. This mark may indicate that you have chosen an inappropriate approach.
- (E) Please be more **explicit** here. The labeled work is insufficiently explicit / insufficiently precise.
- (IC) This argument or work is **incomplete**.
- (IS) The argument you have presented is **insufficient** to prove the conclusion you claim.
- (LN) This **conflates logical and numerical** expressions, variables, or values.

  For example, "$x/6 = 3$" is a logical expression; it is the (possibly true, or possibly false) statement that x divided by 6 is 3. "$x/6$", on the other hand, is a numerical expression; it is a thing that evaluates to a number (or would, once x were given a value). Logical expressions may be combined with logical operators like "and" and "implies". Numerical expressions may not be; it is nonsense to say, for example, that 5 implies 7.
- (M) Use formal **mathematical** notation here, and/or express what you mean in mathematical terms.
- (U) Please submit only one answer to each question.
- (UC) This is **unclear** or confusing. Make your reasoning or instructions more clear or straightforward.
- (V) There is a problem here with your proof/argument: it is **invalid**, insufficiently rigorous, or otherwise not convincing. This may be because there are holes in your logic, because you made an invalid inference or equivalence, because your conclusions are not supported by your argument, because you have made a significant claim without proof, because you have failed to address an important possibility or case, or for some other reason.

Question 1

(a) **isMother**, **isTaller**

(b) **isTaller**

(c) **isMother**

(d) **isTaller**

(e) **isMother**, **isTaller**

Question 2

(a) $\neg[(\neg p \wedge \neg q) \vee (p \vee q)] \equiv \neg(\neg p \wedge \neg q) \wedge \neg(p \vee q)$
$$\equiv (p \vee q) \wedge \neg(p \vee q)$$
$$\equiv \text{False}$$

(b) $\neg(p \Rightarrow r) \vee \Big(p \wedge \big[(q \wedge r) \vee \neg(r \vee \neg q)\big]\Big) \equiv \neg(\neg p \vee r) \vee \Big(p \wedge \big[(q \wedge r) \vee (q \wedge \neg r)\big]\Big)$
$$\equiv (p \wedge \neg r) \vee \Big(p \wedge \big[q \wedge (r \vee \neg r)\big]\Big)$$
$$\equiv (p \wedge \neg r) \vee (p \wedge [q \wedge \text{True}])$$
$$\equiv (p \wedge \neg r) \vee (p \wedge q)$$
$$\equiv p \wedge (\neg r \vee q)$$
$$\equiv p \wedge (r \Rightarrow q)$$

(2.I) The question asked you to simplify the proposition, not to state properties of it (*e.g.* satisfiability).

Question 3

(a)
- For $n = 1$, $n^3 + 2n = 1 + 2 = 3$, which is divisible by 3.
- Assume $k^3 + 2k$ is divisible by 3, for some integer k.

$$\text{Then } (k+1)^3 + 2(k+1) = k^3 + 3k^2 + 3k + 1 + 2k + 2$$
$$= [k^3 + 2k] + 3(k^2 + k + 1)$$
$$= 3x + 3y \text{ for some integers x and y}$$
$$= 3(x + y), \text{ which is divisible by 3.}$$

By the principle of mathematical induction, $n^3 + 2n$ is divisible by 3 for all $n \geq 1$.

(b) This sum is $n^2 + (n+1)^2$ for any $n \geq 1$.
$$n^2 + (n+1)^2 = n^2 + n^2 + 2n + 1$$
$$= 2(n^2 + n) + 1$$
$$= 2x + 1 \text{ for some integer } x,$$
so the sum is odd.

(3.I) The marked text is *something you want to prove*. You can't just state it; only once you prove it may you state that it is the case.

(In particular, note that the general goal of the inductive step in a proof by induction is to prove that $P(k) \Rightarrow P(k+1)$ for some predicate P. In order to prove this, you begin by assuming that the *inductive hypothesis*, $P(k)$, is true, and then you prove that $P(k+1)$ follows from that. "$P(k) \Rightarrow P(k+1)$" is something you want to prove; you don't get to just state it.)

Question 4

(a) **Invalid.** The cases presented in this proof address only *odd* integers n. Nothing has been proven about even integers, but the statement was about *all* positive integers.

(b) **Valid.** (All that is required to prove the statement false is to demonstrate a single counterexample: a number that is not a whole power of a prime, and is not within 2 of any number that is a power of a prime. Showing that all numbers within 2 of the counterexample value are not powers of primes suffices to demonstrate this.)

(c) **Invalid.** Anything implies True. Deriving True from a hypothesis tells us nothing about the truth value of that hypothesis.