1. [4.36] **Claim:** $x, y \in \mathbb{Q} \Rightarrow x - y \in \mathbb{Q}$.
   Proof: If x and y are both rational, then ...

2. [4.38]

   - The claim that "if xy and x are both rational, then y is too" is **Tralse**. **Proof:** ...
   - The claim that "if $x - y$ and x are both rational, then y is too" is **Falrue**. **Proof:** ...

3. [4.40] **Claim:** For all integers n, $3 \nmid (n^2 + 1)$.
   Proof: Let n be some integer. We proceed by cases, depending on ...

   - Say [...]. Then ...
   - Say [...]. Then ...

   Having exhausted all cases for integers n, we conclude that the claim is true for all integers n.

4. [4.47] **Claim:** for $x, y \in \mathbb{R}^{\geq 0}$, we have $\sqrt{xy} \leq (x + y)/2$.
   Proof: We know that $r^2 \geq 0$ for any real number r, and therefore:

$$\begin{aligned} & (x-y)^2 \geq 0 \\ \Rightarrow \quad & \ldots \geq \ldots \end{aligned}$$

   I can insert a comment in the middle of a derivation like so...

$$\begin{aligned} \Rightarrow \quad & \ldots \geq \ldots \\ \Rightarrow \; & (x+y)/2 \geq \sqrt{xy} \end{aligned}$$

5. [4.48] **Claim:** for $x, y \in \mathbb{R}^{\geq 0}$, $\sqrt{xy} = (x + y)/2 \iff x = y$.
   Proof: We prove both directions of the mutual implication separately:

   - *Sub-claim:* for $x, y \in \mathbb{R}^{\geq 0}$, $\sqrt{xy} = (x + y)/2 \Rightarrow x = y$.
     Proof: ...
   - *Sub-claim:* for $x, y \in \mathbb{R}^{\geq 0}$, $x = y \Rightarrow \sqrt{xy} = (x + y)/2$.
     Proof: ...

6. [4.51] **Claim:** For $n \in \mathbb{Z}^{\geq 0}$, if $n \,\%\, 4 \in \{2, 3\}$, then n is not a perfect square.
   Proof: We prove the contrapositive. If n is a perfect square, then ...

7. [4.56] **Claim:** For $x, y \in \mathbb{R}^{>0}$, if $x^2 - y^2 = 1$, then x or y (or both) is not an integer.
   Proof: Say that $x^2 - y^2 = 1$, and let us assume, for the sake of contradiction, that x and y are both positive integers. Then ... ... this is a contradiction, and therefore x and y cannot both be integers.

8. [4.61] **False claim:** If xy is rational, then x and y are rational.
   Disproof: ...